Diagnosis of a nonlinear system with stabilizing control

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Abstract. In this paper is used the nonlinear systems diagnosis by means of differential algebraic techniques to obtain an estimate of the fault vector. Then, with these estimates of the fault components is obtained a stabilizing control law for the states. this result is illustrated by an example of a nonlinear system with control in one of its states, where the system is affected by the action of two additive faults. It will also be shown by means of numerical simulations that under this type of faults the state stays stable.
Keywords.- *Diagnosis, Stabilizing control law.*

1 Introduction

Systems diagnosis has been studied for more than three decades, see for instance [10]. In [11] a direct extension of the unknown input observer (UIO) results in linear systems to the nonlinear case was considered. An alternative to the nonlinear unknown input observer approach in nonlinear uncertain systems was proposed by Seliger and Frank [9], where the presence of modelling uncertainties is not taken into account, however, the reader is referred to the works of Diop and Martínez-Guerra [1], [2], where the presence of uncertainties is included using this methodology. For instance, [1], [2] presents an algebraic approach to solve the diagnosis problem. It consists on translating the solvability of the problem in terms of the algebraic observability of the variable which models the *fault*. The framework in which this paper is conceived is based essentially in the language of differential algebra. In [4], [5], [8], the methodologies employed for the observer design only include full order observers without considering uncertainty estimation, however, in this communication, the fault dynamics is considered as an uncertainty. In the proposed procedure, it is not necessary the construction of a full order observer, instead, a reduced order uncertainty observer is constructed using differential algebraic techniques applied to the fault estimation in the diagnosis problem.
The *main results* of this paper are: the estimation of the faults using differential algebraic techniques and the construction of a stabilizing control law for the

states depending on the fault estimates, this result is illustrated by an example of a nonlinear system with control in one of its states, where the system is affected by the action of two additive faults. It will also be shown by means of numerical simulations that under this type of faults the system stays stable.
The rest of this paper is organized as follows: in Section 2 some basic definitions on observability and systems diagnosability in a differential algebraic framework are introduced. Statement of the problem and the diagnosability condition are described in Section 3. In Section 4 an example with the application of the proposed methodology is shown. In Section 5, the construction of a reducer order uncertainty observer is described. In section 6, we show the numerical results. Finally, in Section 7 we will close the paper with some concluding remarks.

2 Basic Definitions

Before starting, some differential algebra definitions are introduced [3], [4], [6].

Definition 1. *A differential field extension L/k is given by two differential fields k and L, such that: i) k is a subfield of L, ii) the derivation of k is the restriction to k of the derivation of L.*
Example $\mathbb{Q}$,$\mathbb{R}$ *and* $\mathbb{C}$ *are differential field extensions, where* $\mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$.

Definition 2. *An element is said to be differentially algebraic with respect to the field k if it satisfies a differential algebraic equation with coefficients over k.*
Example $\mathbb{R}\langle e^{at}\rangle/\mathbb{R}$ *is a differential field extension* $\mathbb{R} \subset \mathbb{R}\langle e^{at}\rangle$, $x=e^{at}$ *is a solution of* $P(x)=\dot{x}\text{-}ax=0$ *(a is a constant).*

Definition 3. *An element is said to be differentially transcendental over k, if and only if, it is not differentially algebraic over k.*

Definition 4. *A dynamics is a finitely generated differential algebraic extension $G/k\langle u\rangle$ ($k\langle u,\xi\rangle$,$\xi \in G$). Any element of G satisfies an algebraic differential equation with coefficients being rational functions over k in the elements of u and a finite number of their time derivatives.*
Example Let consider the input-output system $\ddot{y} + \omega^2 sen\,(y) = u$, *equivalent to the system:*

$$\Sigma_B \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\omega^2 sen(x_1) + u \\ y \ = x_1 \end{cases} \tag{1}$$

System (1) is a dynamics of the form $\mathbb{R}\langle u,y\rangle/\mathbb{R}\langle u\rangle$ where $G = \mathbb{R}\langle u,y\rangle$, $y \in G$ and $k = \mathbb{R}$. Any solution of (1) satisfies the following algebraic differential equation:

$$\left(y^{(3)} - \dot{u}\right)^2 + \left(\dot{y}\left(y^{(2)} - u\right)\right)^2 = (\omega\dot{y})^2.$$

Definition 5. *Let a subset $\{u,y\}$ of G in a dynamics $G/k\langle u\rangle$. An element in G is said to be algebraically observable with respect to $\{u,y\}$ if it is algebraic over $k\langle u,y\rangle$. Therefore, a state x is said to be algebraically observable if, and only if, it is algebraically observable with respect to $\{u,y\}$. A dynamics $G/k\langle u\rangle$, with output y in G is said to be algebraically observable if, and only if, all its states have this property.*

Example System Σ_B in (1) with output $y \in \mathbb{R}\langle u,y\rangle$ is algebraically observable, since x_1 and x_2 satisfies two algebraic differential polynomials with coefficients in $\mathbb{R}\langle u,y\rangle$, i.e.

$$x_1 - y = 0$$
$$x_2 - \dot{y} = 0.$$

3 Statement of the Problem

Let consider the class of nonlinear systems described by [2], [6]:

$$\begin{array}{l} \dot{x}(t) = A(x,\bar{u}) \\ y(t) = h(x,u) \end{array} \tag{2}$$

Where $x = (x_1,...,x_n)^T \in R^n$ is a state vector, $\bar{u} = (u,f) = (u_1,...,u_{m-\mu}, f_1,...,f_\mu) \in R^{m-\mu} \times R^\mu$ where u is the stabilizing control (a known input vector) and f is an unknown fault vector, $y = (y_1,...,y_p) \in R^p$ is the output, A and h are assumed to be known analytical vector functions.
Then, the problem is to estimate the fault vector to obtain a stabilizing control depending on these fault estimates in order to stabilize the state vector.

Definition 6. *Given a fault f, it is called algebraically observable if each component f_i of the fault is algebraic over the differential field $k\langle u,y\rangle$.*

Definition 7. *An element $f \in k\langle\bar{u},y\rangle$ is said to be algebraically observable if f satisfies a differential algebraic equation with coefficients over $k\langle u,y\rangle$.*

Definition 8. *The class of nonlinear systems described by (2) is said diagnosable if it is possible to estimate the fault f from the system equations and the time histories of the data u and y, i. e., it is diagnosable if f is algebraically observable with respect to u and y.*

Remark It was already pointed out [2] that a diagnosable system need not to be observable, and vice versa. Indeed, the following system

$$\begin{cases} \dot{x}_1 = -x_1 + x_2, \\ \dot{x}_2 = x_2 + u + f, \\ y = \quad x_2, \end{cases} \tag{3}$$

is diagnosable, i. e. $f = \dot{y} - y - u$, but it is not observable since x_1 is not observable with respect to u and y.

4 Example

In the following paragraphs is used the proposed methodology to obtain the diagnosability conditions of the fault components.
Consider the following nonlinear system

$$\begin{aligned} \dot{x}_1 &= -x_1 + f_1 x_2^3 + f_2 x_2 x_3 + u \\ \dot{x}_2 &= x_3 + f_1 \\ \dot{x}_3 &= -x_2^3 + f_2 \\ y_1 &= x_2 \\ y_2 &= x_3 \end{aligned} \tag{4}$$

where the control u can be naturally expressed as

$$u = -\hat{f}_1 x_2^3 - \hat{f}_2 x_2 x_3$$

and replacing u in $\dot{x}_1$ is obtained

$$\dot{x}_1 = -x_1 + \left(f_1 - \hat{f}_1\right) x_2^3 + \left(f_2 - \hat{f}_2\right) x_2 x_3 \tag{5}$$

It is observed that if the second and third term of the right of the equation (5) become equal to zero, then the state x_1 will be stable. With this purpose, it is necessary to obtain an estimate of the fault components and this is achieved imposing certain conditions, which will be presented in the observer synthesis. By replacing y_1 and y_2 in (4) it is not hard to obtain

$$\begin{aligned} \dot{y}_1 - y_2 - f_1 &= 0 \\ \dot{y}_2 + y_1^3 - f_2 &= 0 \end{aligned} \tag{6}$$

Then, system (4) is diagnosable and the fault components satisfies the following algebraic equations over $k\langle u, y\rangle$:

$$\begin{aligned} f_1 &= \dot{y}_1 - y_2 \\ f_2 &= \dot{y}_2 + y_1^3 \end{aligned} \tag{7}$$

where this equations are called the diagnosability conditions for f_1 and f_2.

5 Observer Synthesis

Once that we have the diagnosability conditions of the faults it is necessary to propose the construction of an observer, to obtain by means of numerical simulations the estimates of these faults, as follows.

Let consider system (2). The fault vector f is unknown and it be assimilated as a state with uncertain dynamics. Then, to estimate it the state vector is extended to deal with the unknown fault vector. The new extended system is given by

$$\begin{aligned} \dot{x}(t) &= A(x, \bar{u}) \\ \dot{f} &= \Omega(x, \bar{u}) \\ y(t) &= h(x, u) \end{aligned} \tag{8}$$

where $\Omega(x, \bar{u})$ is a bounded uncertain function.

The following hypotheses are assumed:
H1: $\Omega(x, \bar{u})$ is bounded, i.e., $|\Omega(x, \bar{u})| \leq M$.
H2: $f(t)$ is algebraically observable over $k\langle u, y\rangle$.
H3: γ is a $\mathbf{C}^1$ real-valued function.

Next Lemma describes the construction of a proportional reduced order observer for (8).
Lemma 2 The system

$$\dot{\hat{f}} = K\left(f - \hat{f}\right) \tag{9}$$

is an asymptotic reduced order observer for system (8), where $\hat{f}$ denotes the estimate of f, f is given by its algebraic equation with coefficients in $k<u.y>$ and $K \in \mathbb{R}^+$ determines the desired convergence rate of the observer, if the following assumption is satisfied:

H4: $\left|e^{-\int K dt}\right| = 0$ with t_0 sufficiently large and $\lim\limits_{t \to t_0} \sup \frac{M}{|K|} = 0$.

Sometimes, the output time derivatives (which are unknown), appear in the algebraic equation of the fault, then, it is necessary to use an auxiliary variable to avoid using them.

Corollary The dynamic system (9) along with

$$\dot{\gamma} = \psi(x, \bar{u}, \gamma), \text{ with } \gamma_0 = \gamma(0) \text{ and } \gamma \in \mathbf{C}^1 \tag{10}$$

constitute a proportional asymptotic reduced order fault observer for the system (8), where $\gamma \in \mathbf{C}^1$ is a change of variable which depends on the estimated fault $\hat{f}$, and the states variables.

Further details can be found in [7].

Remark It should be noted that f in (9) is obtained from the algebraic observability condition, that is to say, f is replaced in (9) by its algebraic equation in

$k\langle u, y\rangle$.

The performance of the reduced order observer estimator is shown by means of numerical simulations.

6 Numerical results

The following equation represents the uncertainty dynamics of the fault components:

$$\dot{f}(t) = \Omega(x, \bar{u})$$

In order to estimate the fault variable f, the following reduced order observer is proposed:

$$\begin{aligned} \dot{\hat{f}}_1 &= K_1\left(f_1 - \hat{f}_1\right) \\ \dot{\hat{f}}_2 &= K_2\left(f_2 - \hat{f}_2\right) \end{aligned} \tag{11}$$

then, from (7) and (11) is obtained:

$$\begin{aligned} \dot{\hat{f}}_1 &= K_1\left(\dot{y}_1 - y_2 - \hat{f}_1\right) \\ \dot{\hat{f}}_2 &= K_2\left(\dot{y}_2 + y_1^3 - \hat{f}_2\right) \end{aligned}$$

Note that $\dot{y}_1$ and $\dot{y}_2$ are not available. However, the following auxiliary variables allows to circumvent this problem. Define

$$\begin{aligned} \gamma_1 &= \hat{f}_1 - K_1 y_1 \\ \gamma_2 &= \hat{f}_2 - K_2 y_2 \end{aligned}$$

Then, the reduced order observer is given by

$$\begin{aligned} \dot{\gamma}_1 &= -K_1\left(y_2 + \gamma_1 + K_1 y_1\right) \\ \dot{\gamma}_2 &= K_2\left(y_1^3 - \gamma_2 - K_2 y_2\right) \\ \hat{f}_1 &= \gamma_1 + K_1 y_1 \\ \hat{f}_2 &= \gamma_2 + K_2 y_2 \end{aligned} \tag{12}$$

where $\gamma_1, \gamma_2 \in \mathbb{C}^1$

The simulation results are obtained with initial conditions $\gamma_1=\gamma_2 = 0$ and $K_1 = 5$, $K_2 = 2$. In figure 1 the estimates of the faults which converges to the current faults are shown. Figure 2 describes the numerical simulations corresponding to the state $x_1$without stabilizing control and the stable dynamic of state x_1 under the action of the stabilizing control law.

7 Concluding Remarks

In this paper was presented a stabilizing control for the states depending on the fault estimates, this result is illustrated by an example of a nonlinear system with control in one of its states, where the system is affected by the action of two additive faults. It is also shown by means of numerical simulations that under this type of faults the state x_1 remains stable.

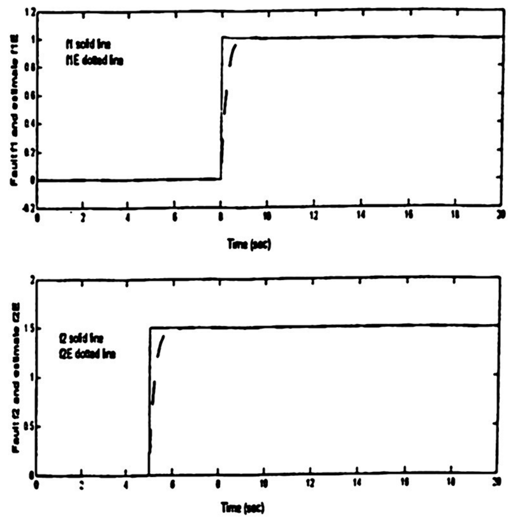

Fig. 1. True dynamics of the fault steps at times 5 and 8 seconds in solid line and its estimates in dotted line.

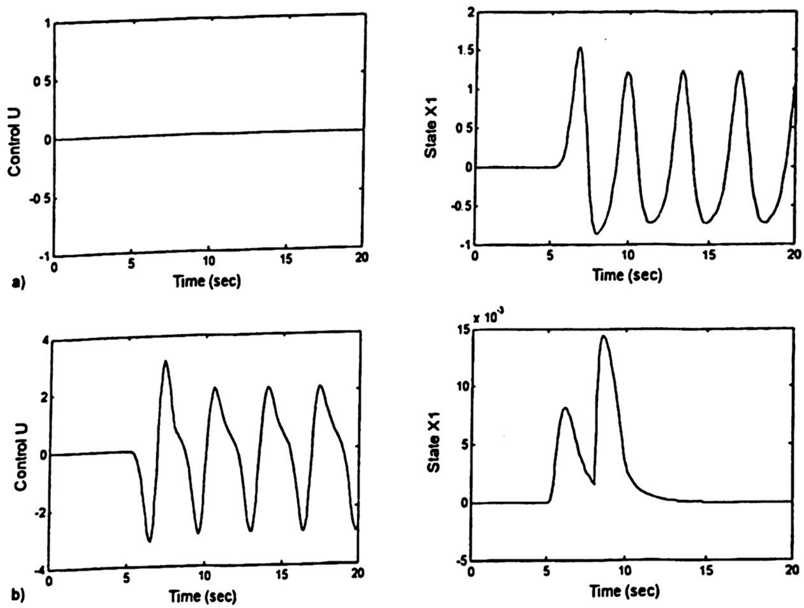

Fig. 2. a) Simulation results of state x_1 without control. b) Numerical results of the state x_1 under the action of the stabilizing control u.

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